The bracketed secant method

1. Use three steps of the bracketed secant method to approximate a root of the function $f(x) \stackrel{\text{def}}{=} \frac{\sin(x)}{x} + e^{-x}$ starting with $a_0 = 3.0$ and $b_0 = 4.0$.

Answer: To ten significant digits, we have [3, 4], [3, 3.361683641], [3, 3.275551379], [3, 3.267328591]

2. Use three steps of the bracketed secant method to approximate a root of the function $f(x) \stackrel{\text{def}}{=} x^3 - 3x + 1$ starting with $a_0 = 1$ and $b_0 = 2$.

Answer: [1, 2], [1.25, 2], [1.407407407, 2], [1.482366864, 2].

3. If you continue to iterate the bracketed bisection method in Question 2, what root does it converge to?

Answer: To ten significant digits, 1.532088886

4. If you repeat Question 2, but alternate between using the bracketed secant method and the bisection method, does the technique appear to converge more quickly or more slowly?

Answer: More quickly: [1, 2], [1.25, 2], [1.25, 1.625], [1.496376812, 1.625]

5. In general, should you apply the bracketed secant method if you don't already have an idea as to what a root of a function is?

Answer: The answer is perhaps. For the bracketed secant method, you must already have a bracket of the root, although the function may also have a discontinuity instead of a root.

6. The function x^2 has a double root at $x = 0$. Can you apply the bracketed secant method to find a double root?

Answer: No, for the function has the same sign in the vicinity of a root, so it is impossible to bracket it. Also, whereas x^2 has a double root at $x = 0$, the function $x^2 + \varepsilon$ has no real roots for $\varepsilon > 0$.